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# Recovering information of tunneling spectrum from weakly isolated horizon

**Ge-Rui Chen**[a], **Yong-Chang Huang**

Institute of Theoretical Physics, Beijing University of Technology, Beijing 100124, China

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**Abstract** In this paper we investigate the properties of tunneling spectrum from weakly isolated horizon (WIH)—a locally defined black hole. We find that there exist correlations among Hawking radiations from a WIH, information can be carried out by such correlations, and the radiation is an entropy conservation process. Through revisiting the calculation of the tunneling spectrum from a WIH, we find that Zhang et al.'s (Ann Phys 326:350, 2011) requirement that radiated particles have the same angular momenta of a unit mass as that of the black hole is unnecessary, and the energy and angular momenta of the emitted particles are very arbitrary, restricted only by keeping the cosmic censorship hypothesis of black holes. So we resolve the information loss paradox based on the method of Zhang et al. (Phys Lett B 675:98, 2009; Ann Phys 326:350, 2011; Int J Mod Phys D 22:1341014, 2013) in a general case.

## 1 Introduction

Stephen Hawking' s astounding discovery that black holes radiate a black body spectrum [1,2] has been greatly stimulating the development of the theory of the black hole. Hawking radiation gives us new insights into gravitational physics and also provides some hints of quantum gravity. From Hawking's famous work, we know that black holes are not the final states of stars, and, with the emission of Hawking radiation, they could lose energy, shrink, and eventually evaporate completely. However, because of the nature of the purely thermal spectrum, it causes a disturbing and difficult problem: what happens to information during black hole evaporation? This scenario is inconsistent with the unitary principle of quantum mechanics [3–6]. Around the year 2000, Parikh and Wilczek, contemplating Hawking's heuristic picture of tunneling triggered by vacuum fluctuations near the horizon, proposed a semiclassical method to investigate the emission rate by treating Hawking radiation as a tunneling process [7,8]. This method considers the back reaction of the emitted particle to the spacetime, and it does not fix the background spacetime. They found that the barrier of tunneling is created by the outgoing particle itself, and when energy conservation is considered, a non-thermal spectrum is given, which supports the underlying unitary theory.

From the year 2009, Refs. [9–11] discussed in detail Parikh and Wilczek's non-thermal spectrum. They found that there exist correlations among Hawking radiations (of tunneled particles) from a black hole, the correlations are equal to mutual information, and black hole radiation is an entropy conservation process, which is consistent with unitarity of quantum mechanics. Their discussions are based on stationary black holes. The stationary black hole is represented by an event horizon. However, this representation of black holes possesses some drawbacks [12]. Firstly, to find the event horizon knowledge of the metric of the entire spacetime is required, in other words, defining a black hole by local conditions is more reasonable. Secondly, the event horizons are determined by the exact solutions of the Einstein equation (most of which are of high symmetry), while realistic black holes are often distorted by the gravitational interaction with the matter and radiation around them. Finally, the systematic development of the black hole thermodynamics requires the description involving local information rather than the data describing distant regions. The close relationship between Hawking radiation and black hole thermodynamics reveals that it is better to study the properties of Hawking radiation in the framework of locally defined black holes. A successful example of a locally defined black hole is the theory of WIHs [14–18] first proposed by Ashtekar and developed by many others. For the black hole information paradox, it is important to study the problem for a WIH, since this model describes real black holes and already includes all the stationary cases. It is the first time to investigate the black hole information paradox of Hawking radiation from the viewpoint of locally defined black holes. We prove that for a

[a] e-mail: chengerui@emails.bjut.edu.cn

WIH, there exist correlations among Hawking radiations; information can be carried out by such correlations, and the radiation is an entropy conservation process. Furthermore, in our analysis Ref. [10]'s requirement that radiated particles have the same angular momenta of a unit mass as that of the black hole, $j = \frac{J}{E}\varepsilon$, is unnecessary, and the energy and angular momenta of emitted particles are very arbitrary, restricted only by keeping the cosmic censorship hypothesis of the black holes.

This paper is organized as follows. In Sect. 2, we revisit the tunneling method to get the non-thermal spectrum of weakly isolated horizon. In Sect. 3, we investigate the properties of this non-thermal spectrum. In the last section, we give some discussions and conclusions.

## 2 Revisiting Parikh and Wilczek' s tunneling spectrum from weakly isolated horizon

In this section we revisit the calculation of the tunneling spectrum of a WIH with some differences from the original discussion [19], and strictly follow Parikh and Wilczek's calculation [7,8].

Reference [17] established the first law of WIH thermodynamics,

$$\delta E = \frac{1}{8\pi}\kappa\delta A + \Omega\delta J. \tag{1}$$

The expressions of the surface gravity, angular velocity, and horizon energy of a WIH are given by

$$\kappa = \frac{R^4 - 4J^2}{2R^3\sqrt{R^4+4J^2}}, \quad \Omega = \frac{2J}{R\sqrt{R^4+4J^2}},$$
$$E = \frac{\sqrt{R^4+4J^2}}{2R}, \tag{2}$$

where $R$ is the horizon radius, defined as

$$R \equiv \sqrt{\frac{A}{4\pi}}. \tag{3}$$

$A$ is the area of any cross section of the horizon, so the entropy of a WIH is

$$S = \frac{A}{4} = \pi R^2. \tag{4}$$

In the semiclassical limit, we can apply the WKB formula. The emission rate $\Gamma$ is given as

$$\Gamma \sim \exp(-2Im\ I), \tag{5}$$

where $I$ is the action of the emitted particle. The imaginary part of the action for an s-wave outgoing positive energy particle, from $r_{in}$ to $r_{out}$, can be calculated as

$$Im\ I = Im\int_{r_{in}}^{r_{out}} p_r dr = Im\int_{r_{in}}^{r_{out}}\int_0^{p_r} dp_r dr. \tag{6}$$

From Hamilton' s equation of the emitted particle,

$$dp_r = \frac{dH}{\dot{r}}, \tag{7}$$

where $H = \varepsilon$ is the energy of the emitted particles, we get

$$Im\ I = Im\int_{r_{in}}^{r_{out}}\int_0^{\omega}\frac{d\varepsilon}{\dot{r}}dr. \tag{8}$$

From Ref. [19], the outgoing null geodesic is

$$\dot{r} = B_t(\varepsilon+\bar{\varepsilon})r + O(r^2) = \kappa r + O(r^2), \tag{9}$$

where $\kappa = B_t(\varepsilon+\bar{\varepsilon})$ is the surface gravity of the horizon, and it is constant on the horizon [17]. So the imaginary part of the action is

$$\begin{aligned} Im\ I &= Im\int_{r_{in}}^{r_{out}}\int_0^{\omega}\frac{d\varepsilon}{\kappa r + O(r^2)}dr \\ &= Im\int_0^{\omega}\int_{r_{in}}^{r_{out}}\frac{dr}{\kappa r + O(r^2)}d\varepsilon \\ &= Im\int_0^{\omega}\left[\pi i\frac{1}{\kappa}\right]d\varepsilon = \pi\int_0^{\omega}\frac{d\varepsilon}{\kappa}, \end{aligned} \tag{10}$$

where the integral of $r$ is done by deforming the contour around the pole in the third equality. For a non-rotating WIH, $\Omega = 0$ and $J = 0$, so we obtain from Eqs. (2)

$$\kappa = \frac{1}{4E}. \tag{11}$$

We fix the total mass of the spacetime, and allow the black hole mass to fluctuate. After emitting a particle with energy $\varepsilon$ the black hole mass becomes $E-\varepsilon$, so we obtain

$$\begin{aligned} Im\ I &= \pi\int_0^{\omega}\frac{d\varepsilon}{\kappa'} = \pi\int_0^{\omega}4E'd\varepsilon = \pi\int_0^{\omega}4(E-\varepsilon)d\varepsilon \\ &= 4\pi\varepsilon\left(E-\frac{\varepsilon}{2}\right). \end{aligned} \tag{12}$$

According to Eqs. (2) and (4), the entropy of the non-rotating WIH can be expressed as

$$S = \pi R^2 = 4\pi E^2, \tag{13}$$

and the change of the entropy after emitting a particle is

$$\Delta S = 4\pi[(E-\varepsilon)^2 - E^2] = -8\pi\varepsilon\left(E-\frac{\varepsilon}{2}\right). \tag{14}$$

So the tunneling rate is

$$\begin{aligned} \Gamma &= \exp(-2Im\ I) = \exp\left[-8\pi\varepsilon\left(E-\frac{\varepsilon}{2}\right)\right] = \exp(\Delta S) \\ &= \exp(4\pi[(E-\varepsilon)^2 - E^2]). \end{aligned} \tag{15}$$

Next, we discuss the rotating WIH. From Eqs. (2), (3) and (4), we can rewrite angular velocity $\Omega$, surface gravity $\kappa$ and entropy $S$ as

$$\Omega = \frac{J}{2E(E^2+\sqrt{E^4-J^2})}, \quad \kappa = \frac{\sqrt{E^4-J^2}}{2E(\sqrt{E^4-J^2}+E^2)},$$
$$S = \pi R^2 = 2\pi(E^2+\sqrt{E^4-J^2}). \tag{16}$$

For axial symmetric WIH, using the formula [19,20], the imaginary part of the action $I$ should be

$$\begin{aligned} Im\ I &= Im\int[p_r dr - p_\phi d\phi] = Im\int\left[p_r - \frac{p_\phi\dot{\phi}}{\dot{r}}\right]dr \\ &= Im\int\int\frac{dH-\dot{\phi}dp_\phi}{\dot{r}}dr \\ &= Im\int\int\frac{d\varepsilon-\Omega dj}{\dot{r}}dr = Im\int\int\frac{dr}{\dot{r}}(d\varepsilon-\Omega dj) \\ &= Im\int\frac{\pi i}{\kappa}(d\varepsilon-\Omega dj) = \pi\int\frac{d\varepsilon-\Omega dj}{\kappa}, \end{aligned} \tag{17}$$

where $\Omega$ is the angular velocity of the horizon and $j = p_\phi$ is the angular momentum of the emitted particle. We consider the s-wave, so the particles radiate along the normal direction of the horizon, that is, $\dot{\phi} = \Omega$. This is the requirement for the emitted particles, and the condition that particles be emitted with the original angular momentum of a unit mass of the black hole, $j = \frac{J}{E}\varepsilon$, is unnecessary (see Ref. [10]).

When the particles' self-gravitation is taken into account we should replace $E$ and $J$ with $E-\varepsilon$ and $J-j$ in the expression of $\kappa$ and $\Omega$, so we get from Eqs. (17),

$$\begin{aligned} Im\ I &= \pi\int\frac{d\varepsilon - \frac{J-j}{2(E-\varepsilon)[(E-\varepsilon)^2+\sqrt{(E-\varepsilon)^4-(J-j)^2}]}dj}{\frac{\sqrt{(E-\varepsilon)^4-(J-j)^2}}{2(E-\varepsilon)[\sqrt{(E-\varepsilon)^4-(J-\varepsilon)^2}+(E-\varepsilon)^2]}} \\ &= \pi\int\frac{2(E-\varepsilon)[\sqrt{(E-\varepsilon)^4-(J-\varepsilon)^2}+(E-\varepsilon)^2]}{\sqrt{(E-\varepsilon)^4-(J-\varepsilon)^2}}d\varepsilon \\ &\quad -\frac{J-j}{\sqrt{(E-\varepsilon)^4-(J-\varepsilon)^2}}dj. \end{aligned} \tag{18}$$

We do not need to do the integration directly. The change of the back hole entropy after emitting a particle is

$$\begin{aligned} \Delta S &= 2\pi[(E-\varepsilon)^2+\sqrt{(E-\varepsilon)^4-(J-j)^2}] \\ &\quad -2\pi[E^2+\sqrt{E^4-J^2}], \end{aligned} \tag{19}$$

and it is easy to get

$$\begin{aligned} \frac{\partial(\Delta S)}{\partial\varepsilon} &= -4\pi\left[\frac{(E-\varepsilon)^3}{\sqrt{(E-\varepsilon)^4-(J-j)^2}}+(E-\varepsilon)\right], \\ \frac{\partial(\Delta S)}{\partial j} &= 2\pi\frac{J-j}{\sqrt{(E-\varepsilon)^4-(J-j)^2}}. \end{aligned} \tag{20}$$

By putting the above equations into Eq. (18), we have

$$ImS = -\frac{1}{2}\left[\int\frac{\partial(\Delta S)}{\partial\varepsilon}d\varepsilon + \frac{\partial(\Delta S)}{\partial j}dj\right] = -\frac{1}{2}\Delta S. \tag{21}$$

So the tunneling rate is

$$\begin{aligned} \Gamma &= \exp(-2ImS) = \exp(\Delta S) \\ &= \exp(2\pi[(E-\varepsilon)^2+\sqrt{(E-\varepsilon)^4-(J-j)^2}] \\ &\quad -2\pi[E^2+\sqrt{E^4-J^2}]). \end{aligned} \tag{22}$$

Our next discussion is based on this equation.

## 3 Information recovery of tunneling spectrum from weakly isolated horizon

In this section, we investigate the properties of the tunneling spectrum from a WIH following Refs. [9–11]. From Eq. (22), the probability for the emission of a particle with energy and angular momentum $(\varepsilon_1, j_1)$ is

$$\begin{aligned} \Gamma(\varepsilon_1, j_1) &= \exp(2\pi[(E-\varepsilon_1)^2+\sqrt{(E-\varepsilon_1)^4-(J-j_1)^2}] \\ &\quad -2\pi[E^2+\sqrt{E^4-J^2}]). \end{aligned} \tag{23}$$

And the probability for the emission of a particle with energy and angular momentum $(\varepsilon_2, j_2)$ is

$$\begin{aligned} \Gamma(\varepsilon_2, j_2) &= \exp(2\pi[(E-\varepsilon_2)^2+\sqrt{(E-\varepsilon_2)^4-(J-j_2)^2}] \\ &\quad -2\pi[E^2+\sqrt{E^4-J^2}]). \end{aligned} \tag{24}$$

Note that $(\varepsilon_1, j_1)$ and $(\varepsilon_2, j_2)$ represent two independent emitted particles, so the expressions should have the same form.

Let us consider a process as follows. Firstly one particle with energy and angular momentum $(\varepsilon_1, j_1)$ emits, and then another particle with energy and angular momentum $(\varepsilon_2, j_2)$ radiates. The probability for the emission of the second particle is

$$\begin{aligned} \Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1) &= \exp(2\pi[(E-\varepsilon_1-\varepsilon_2)^2 \\ &\quad +\sqrt{(E-\varepsilon_1-\varepsilon_2)^4-(J-j_1-j_2)^2} \\ &\quad -2\pi[(E-\varepsilon_1)^2+\sqrt{(E-\varepsilon_1)^4-(J-j_1)^2}]), \end{aligned} \tag{25}$$

which is the conditional probability and is different from the independent probability (24). The probability for two successive emissions with energy and angular momenta $(\varepsilon_1, j_1)$ and $(\varepsilon_2, j_2)$ can be deduced as follows:

$$\begin{aligned} \Gamma(\varepsilon_1, j_1, \varepsilon_2, j_2) &\equiv \Gamma(\varepsilon_1, j_1)\Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1) \\ &= \exp(2\pi[(E-\varepsilon_1)^2+\sqrt{(E-\varepsilon_1)^4-(J-j_1)^2}] \\ &\quad -2\pi[E^2+\sqrt{E^4-J^2}])\exp(2\pi[(E-\varepsilon_1-\varepsilon_2)^2 \\ &\quad +\sqrt{(E-\varepsilon_1-\varepsilon_2)^4-(J-j_1-j_2)^2} \\ &\quad -2\pi[(E-\varepsilon_1)^2+\sqrt{(E-\varepsilon_1)^4-(J-j_1)^2}]) \\ &= \exp(2\pi[(E-\varepsilon_1-\varepsilon_2)^2 \\ &\quad +\sqrt{(E-\varepsilon_1-\varepsilon_2)^4-(J-j_1-j_2)^2} \\ &\quad -2\pi[E^2+\sqrt{E^4-J^2}]). \end{aligned} \tag{26}$$

The last equality is nothing but $\Gamma(\varepsilon_1+\varepsilon_2, j_1+j_2)$, so we get

$$\Gamma(\varepsilon_1, j_1, \varepsilon_2, j_2) \equiv \Gamma(\varepsilon_1, j_1)\Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1) = \Gamma(\varepsilon_1+\varepsilon_2, j_1+j_2). \tag{27}$$

This is an important relationship which tells us that the probability of two particles emitted successively with energy and angular momenta $(\varepsilon_1, j_1)$ and $(\varepsilon_2, j_2)$ is the same as the probability of one particle with energy and angular momentum $(\varepsilon_1+\varepsilon_2, j_1+j_2)$. It is easy to see that

$$\begin{aligned}\Gamma(\varepsilon_1, j_1, \varepsilon_2, j_2, \ldots, \varepsilon_i, j_i) &= \Gamma(\varepsilon_1, j_1)\Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1)\\ &\quad\times\cdots\times\Gamma(\varepsilon_i, j_i|\varepsilon_1, j_1, \ldots, \varepsilon_{i-1}, j_{i-1})\\ &= \Gamma(\varepsilon_1+\cdots+\varepsilon_i, j_1+\cdots+j_i),\end{aligned} \tag{28}$$

which is an important relationship we will use later.

The function

$$C(A\cup B; A, B) = \ln\Gamma(A\cup B) - \ln[\Gamma(A)\Gamma(B)] \tag{29}$$

is used to measure the statistical correlation between two events $A$ and $B$. For the Hawking radiation, the correlation between the two sequential emissions [9–11,13] can be calculated as

$$\begin{aligned}&\ln\Gamma(\varepsilon_1+\varepsilon_2, j_1+j_2) - \ln[\Gamma(\varepsilon_1, j_1)\Gamma(\varepsilon_2, j_2)]\\ &\quad= \ln\frac{\Gamma(\varepsilon_1+\varepsilon_2, j_1+j_2)}{\Gamma(\varepsilon_1, j_1)\Gamma(\varepsilon_2, j_2)}\\ &\quad= \ln\frac{\Gamma(\varepsilon_1, j_1)\Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1)}{\Gamma(\varepsilon_1, j_1)\Gamma(\varepsilon_2, j_2)}\\ &\quad= \ln\frac{\Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1)}{\Gamma(\varepsilon_2, j_2)} \neq 0,\end{aligned} \tag{30}$$

which shows that the two emissions are statistically dependent, and there exist correlations between sequential Hawking radiations from WIH.

Like Eq. (25), the conditional probability $\Gamma(\varepsilon_i, j_i|\varepsilon_1, j_1, \ldots, \varepsilon_{i-1}, j_{i-1})$ is the tunneling probability of a particle being emitted with energy and angular momentum $(\varepsilon_i, j_i)$ after a sequence of radiation from $1 \to (i-1)$. So the conditional entropy taken away by this tunneling particle is then given by

$$\begin{aligned}&S(\varepsilon_i, j_i|\varepsilon_1, j_1, \ldots, \varepsilon_{i-1}, j_{i-1})\\ &\quad= -\ln\Gamma(\varepsilon_i, j_i|\varepsilon_1, j_1, \ldots, \varepsilon_{i-1}, j_{i-1}).\end{aligned} \tag{31}$$

The mutual information for the emission of two particles with energy and angular momenta $(\varepsilon_1, j_1)$ and $(\varepsilon_2, j_2)$ is defined as [9–11]

$$\begin{aligned}S(\varepsilon_2, j_2 : \varepsilon_1, j_1) &\equiv S(\varepsilon_2, j_2) - S(\varepsilon_2, j_2|\varepsilon_1, j_1)\\ &= -\ln\Gamma(\varepsilon_2, j_2) + \ln\Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1)\\ &= \ln\frac{\Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1)}{\Gamma(\varepsilon_2, j_2)},\end{aligned} \tag{32}$$

which shows that mutual information is equal to correlation between the sequential emissions, that is to say, information is carried out by correlations between Hawking radiations.

Let us calculate the entropy carried out by Hawking radiations. The entropy carried out by the first emitted particle with energy and angular momentum $(\varepsilon_1, j_1)$ is

$$S(\varepsilon_1, j_1) = -\ln\Gamma(\varepsilon_1, j_1). \tag{33}$$

The conditional entropy carried out by the second emission after the first emission is

$$S(\varepsilon_2, j_2|\varepsilon_1, j_1) = -\ln\Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1). \tag{34}$$

So the total entropy carried out by the two sequential emissions is

$$S(\varepsilon_1, j_1, \varepsilon_2, j_2) = S(\varepsilon_1, j_1) + S(\varepsilon_2, j_2|\varepsilon_1, j_1). \tag{35}$$

Assuming the black hole is exhausted after radiating $n$ particles, we have the following relationship:

$$\sum_i^n \varepsilon_i = E, \sum_i^n j_i = J, \tag{36}$$

where $E, J$ are the mass and angular momentum of WIH. The entropy carried out by all the emitted particles is

$$\begin{aligned}S(\varepsilon_1, j_1, \ldots, \varepsilon_n, j_n) &= \sum_{i=1}^n S(\varepsilon_i, j_i|\varepsilon_1, j_1, \ldots, \varepsilon_{i-1}, j_{i-1})\\ &= S(\varepsilon_1, j_1) + S(\varepsilon_2, j_2|\varepsilon_1, j_1)\\ &\quad+\cdots+S(\varepsilon_n, j_n|\varepsilon_1, j_1, \ldots, \varepsilon_{n-1}, j_{n-1})\\ &= -\ln\Gamma(\varepsilon_1, j_1) - \ln\Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1) - \cdots\\ &\quad-\ln\Gamma(\varepsilon_n, j_n|\varepsilon_1, j_1, \ldots, \varepsilon_{n-1}, j_{n-1})\\ &= -\ln[\Gamma(\varepsilon_1, j_1)\times\Gamma(\varepsilon_2, j_2|\varepsilon_1, j_1)\\ &\quad\times\cdots\times\Gamma(\varepsilon_n, j_n|\varepsilon_1, j_1, \ldots, \varepsilon_{n-1}, j_{n-1})]\\ &= -\ln\Gamma(\varepsilon_1+\varepsilon_2+\cdots+\varepsilon_n, j_1+j_2+\cdots+j_n)\\ &= -\ln\Gamma(M, J) = 2\pi(E^2+\sqrt{E^4-J^2}) = S_{WIH},\end{aligned} \tag{37}$$

where we use Eq. (28) in the fifth equation and Eq. (16) in the last equation. The result shows that the entropy carried out by all the emitted particles equals the original black hole entropy, so the total entropy is conserved.

## 4 Discussions and conclusions

We have two comments on the above analysis. Firstly, the energy and angular momenta of emitted particles are not arbitrary, because the black hole should satisfy the cosmic censorship hypothesis at any time, that is to say, the black hole should satisfy the relationship, $E^4 \geq J^2$. If the extreme case $E^4 = J^2$ is reached, the radiation will stop since the temperature of the black hole is zero. The sum of the entropy carried out by Hawking radiations and the remaining entropy of the black hole is also conserved. Secondly, Ref. [10] requires that emitted particles have the same angular momentum of a unit mass as that of black hole, $j = \frac{J}{E}\varepsilon$, but we find that this condition is unnecessary in the calculation of Parikh and Wilczek' s tunneling spectrum. For the s-wave, the particles radiate along the normal direction of the horizon, that is, the emitted particles' angular velocity equals the angular velocity of the black hole, which is the requirement for the emitted particles.

In this paper we investigate the information loss paradox of a WIH. We find that for the locally defined black holes, in the tunneling spectrum there exist correlations, and information can be carried out by such correlations, and the entropy is conserved during the radiation process. In our analysis we find that the energy and angular momenta of the emitted particles are very arbitrary, restricted only by keeping the cosmic censorship hypothesis of the black holes. Therefore, we resolve the information loss paradox based on the method of [9–11] against a realistic and general background.

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